

## Angular and Linear Velocity

**Velocity:** The rate at which the location of an object is changing with respect to time.

**Angular Velocity:** The rate at which the angle is changing. If a point is in motion on a circle through an angle of  $\alpha$  radians in time  $t$ , then its angular velocity  $\omega$  is given by  $\omega = \frac{\alpha}{t}$ . Angular velocity is usually expressed as radians per unit of time (radians/hr, radians/min, radians/sec, etc.)

**Examples:**

Convert 650 rpm (revolutions per minute) to radians per minute.  
(Use the fact that 1 revolution =  $2\pi$  radians).

Convert the angular velocity of 1600 rad/hr to rad/sec.

A 24-inch lawnmower blade rotates at a rate of 2000 rpm. What is the angular velocity in radians per second of a point on the tip of the blade?

Find the angular velocity in radians per second for a particle that is moving in a circular path at 4 revolutions per second on a circle of radius 9 ft.

**Linear Velocity:** The rate at which the distance is changing. If a point is in motion on a circle of radius  $r$  through an angle of  $\alpha$  radians in time  $t$ , then its linear velocity  $v$  is given by  $v = \frac{s}{t}$ , where  $s$  is the arc length determined by  $s = \alpha r$ .

**Examples:**

A propeller with a radius of 1.6 meters is rotating at 1500 revolutions per minute. What is the linear velocity in meters per minute for a point on the tip of the propeller?

Find the linear velocity in meters per second for a particle that is moving in a circular path at 7 revolutions per second on a circle of radius 15 meters.

What is the linear velocity in miles per hour of the tip of a 20-inch lawnmower blade that is rotating at 3000 rpms?

Find the linear velocity in miles per hour for a particle that is moving in a circular path at 1800 revolutions per minute on a circle with a diameter of 14 inches.

**Linear Velocity in Terms of Angular Velocity:** If  $v$  is the linear velocity of a point on a circle of radius  $r$ , and  $\omega$  is its angular velocity, then  $v = r\omega$ .

**Example:**

Any point on the surface of the earth (except at the poles) makes one revolution ( $2\pi$  radians) about the axis of the earth in 24 hours. So the angular velocity of a point on the earth is  $2\pi/24$  or  $\pi/12$  radians per hour. The linear velocity of a point on the surface of the earth depends on its distance from the axis of the earth. What is the linear velocity in miles per hour of a point on the equator? (Use 3950 miles as the radius of the earth).